A study of the electromagnetic-fluctuation-induced forces on thin metallic films

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Abstract
Using the plasma model for the metal dielectric function we have calculated
the electromagnetic-fluctuation-induced forces on a free standing metallic film
in vacuum as a function of the film size and the plasma frequency. The force
for unit area is attractive and for a given film thickness it shows an intensity
maximum at a specific plasma value, which cannot be predicted on the basis
of a non-retarded description of the electromagnetic interaction. If the film
is deposited on a substrate or interacts with a plate, both the sign and the
value of the force are modified. It is shown that the force can change the sign
from attraction to repulsion upon changing the substrate plasma frequency. A
detailed comparison between the force on the film boundaries and the force
between film and substrate indicates that, for 50–100 nm thick films, they are
comparable when film–substrate distance is of the order of the film thickness.

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1. Introduction

Electromagnetic-fluctuation-induced forces have been the subject of several investigations
both in the non-retarded small-distance limit (dispersion forces) and in the retarded large-
distance case (Casimir forces) [1–6]. Since the basic work from Lifshitz [1] studies have been
focused mainly on the determination of the forces between two semi-infinite planar media [2]
or between a sphere and a planar medium [7]. Even if model calculations have been performed
for special geometries [5, 6, 8], forces on realistic systems have been obtained starting from
the above mentioned configurations.

For several technological applications, multi-layer systems, obtained by depositing thin
films of different species onto a given substrate, are important and theoretical approaches have
been devised to determine van der Waals forces between laminated media [9, 10]. Casimir

forces between moving parts have been considered as possible source of instabilities in micro- and nano-devices, where the components are in close proximity [11–13]. In many of these systems the situations of interest correspond to interaction between parallel interfaces of films and plates with micro- or submicro-size and submicro-distances. While some of these studies have been performed using simplified models for the interaction, such as the assumption that the interaction is correctly represented by the force between ideal metallic plates, it has been pointed out that a realistic description of adhesion or stiction phenomena has to account for the dependence of the force upon the shape and optical properties of the components [10, 14].

In spite of this large amount of work, a detailed study of the behaviour of the electromagnetic-fluctuation-induced forces in unsupported or deposited conductive films, as a function of their size and optical parameters has not been published. The interest has been focused mainly on the interaction between two-dimensional films, for which forces are supposed to show a peculiar dependence upon the film distance [15–17]. Less interest has been given to the study of the forces on the film boundaries due to vacuum fluctuations, which are present even in an isolated film and depend upon its size and properties.

We can formulate the problem as follows: suppose we have a simple metal, whose dielectric function can be expressed as

$$\epsilon(\omega) = 1 - \frac{\Omega_p^2}{\omega(\omega + i/\tau)}, \tag{1}$$

where Ω_p is the plasma frequency and τ is the relaxation time. If we consider an isolated metal film of thickness d, at $T = 0°$ K we know that, in the limit $\Omega_p \to 0$ the force on the film (the electromagnetic pressure on the film boundaries or the force between them) vanishes and the same happens in the limit of an infinite plasma frequency (perfect metal case). The vanishing of the force is due to the peculiar values of film reflectivity in the two limits, it cannot stay identically zero for physical values of the reflectivity[3]. The problem of what sort of behaviour has the force between these two limits has not been investigated: clearly, for a given film thickness, it must reach at least one maximum of intensity. The questions to be answered are (i) what is the behaviour of the force as a function of Ω_p, in particular at which plasma frequencies are force maxima obtained? (ii) How do such maxima depend upon the film thickness? (iii) How is the behaviour of the force modified when the film is deposited onto a substrate? (iv) How does the electromagnetic force on the free standing film compare with the film–substrate interaction, that may be responsible of adhesion and stiction phenomena?

To provide answers to some of these questions, we have used the Lifshitz approach to the electromagnetic-fluctuation-induced forces to study the force on metallic films. Here we report on some general results that can be obtained using the plasma model dielectric functions. Our purpose is not to achieve a precise description of the force for specific systems, since an accurate evaluation with an intrinsic force uncertainty of few per cent requires a precise determination of the Drude parameters, which are very sensitive to the sample condition [18, 19]. Rather we want to illustrate some general trends that can be understood using a model description of the dielectric properties.

2. Force on isolated metallic films

We consider metallic films of thickness d ranging from 10 nm to a few hundred nm. For typical metallic densities, the electronic distribution deviates significantly from the bulk behaviour

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[3] This can be easily understood by noticing that in the non-retarded regime the force on the free standing metal film is the same as the force between two semi-infinite plates of the same metal separated by a distance equal to the film thickness (see equations (5) and (6) in the text), which is obviously attractive and different from zero.
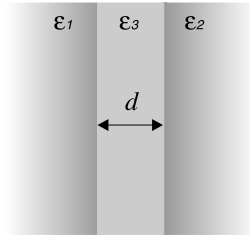
Figure 1. Notation for three-layer system.

when the size of the film is less than about ten times the Fermi wavelength λ_F. Taking $\lambda_F \simeq$ 5 Å, the bulk description is expected to become inaccurate when d is of the order of 50 Å. For such ultrathin film quantum size effects are known to be important [20–24].

We adopt a local description of the electromagnetic properties of the metal based on a dielectric function $\epsilon(\omega)$, i.e. neglecting the wavenumber dependence of the dielectric response. This local theory is expected to be less accurate in thin films than for half space or bulk systems. Recent calculations have shown that non-local corrections to electromagnetic-induced forces for typical metallic densities are of the order of a few tenth of a per cent, suggesting that the local theory can be appropriate in the interpretation of the experimental data [25]. The expression of the force per unit area F at absolute temperature T in a configuration with two semi-infinite planar media of dielectric functions $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$, respectively, separated by a film of thickness d and dielectric function $\epsilon_3(\omega)$ (figure 1) is given by

$$F = -\frac{1}{\pi\beta} \int_0^\infty k \, \mathrm{d}k \sum_n{}' \left\{ \frac{1 - Q_{\mathrm{TM}}(\mathrm{i}\Omega_n)}{Q_{\mathrm{TM}}(\mathrm{i}\Omega_n)} + \frac{1 - Q_{\mathrm{TE}}(\mathrm{i}\Omega_n)}{Q_{\mathrm{TE}}(\mathrm{i}\Omega_n)} \right\} \gamma_3, \tag{2}$$

where k is the modulus of a two-dimensional wave vector parallel to the plates, $\beta = 1/k_B T$, k_B is the Boltzmann constant, $\Omega_n = 2\pi n/\hbar\beta$ is the Matsubara frequency corresponding to the nth thermal fluctuation mode, the prime on the summation indicates that the $n = 0$ term is given half weight. Q_{TM} and Q_{TE} refer to transverse magnetic (TM) and transverse electric (TE) modes, respectively, and are given by

$$Q_{\mathrm{TM}}(\mathrm{i}\omega) = 1 - \frac{(\epsilon_1\gamma_3 - \epsilon_3\gamma_1)(\epsilon_2\gamma_3 - \epsilon_3\gamma_2)}{(\epsilon_1\gamma_3 + \epsilon_3\gamma_1)(\epsilon_2\gamma_3 + \epsilon_3\gamma_2)} \, \mathrm{e}^{-2d\gamma_3} \tag{3a}$$

$$Q_{\mathrm{TE}}(\mathrm{i}\omega) = 1 - \frac{(\gamma_3 - \gamma_1)(\gamma_3 - \gamma_2)}{(\gamma_3 + \gamma_1)(\gamma_3 + \gamma_2)} \, \mathrm{e}^{-2d\gamma_3} \tag{3b}$$

with

$$\gamma_i^2 = k^2 + \frac{\Omega_n^2}{c^2} \epsilon_i(\mathrm{i}\Omega_n) \tag{4}$$

and the dielectric functions are evaluated at the frequency $\mathrm{i}\Omega_n$.

We performed the calculation of the force per unit area on a free standing metal film using equation (2) at $T = 300°$ K, taking $\epsilon_1 = \epsilon_2 = 1$ and $\epsilon_3 = 1 - \Omega_3^2/\omega^2$. We neglect for simplicity relaxation-time effects. Although they are important in determining the infrared response of metals and the calculation we report can be extended to a complex dielectric function, we focus our interest mainly on general trends in the behaviour of electromagnetic-fluctuation-induced forces. For realistic calculations on specific materials relaxation-time effects have to be included and in metal films they can affect the force intensity.
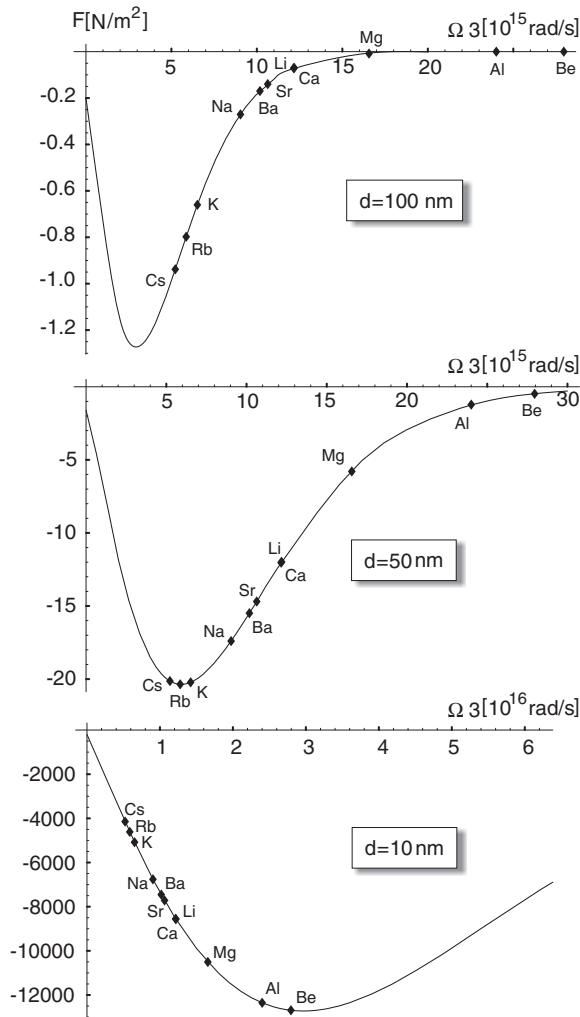
Figure 2. Force as a function of the film plasma frequency: the force increases on decreasing the film thickness d. The calculations were performed summating the first thousand Matsubara frequencies.

Note that there is a basic difference between the electromagnetic-fluctuation-induced forces between two semi-infinite metals and those on the boundaries of a film. This is clearly seen if one considers an ideal (perfectly reflecting) metal, corresponding to an infinite plasma frequency: the interaction between two semi-infinite systems is expressed by the well-known Casimir force $F(d) = \hbar\pi^2 c/240d^4$, while for a film of finite thickness, the force on the boundaries vanishes.

Figure 2 displays the calculated behaviour of F as a function of the plasma frequency for films of different thickness ranging from 10 to 100 nm. Note that at finite temperatures the force vanishes in the large plasma frequency limit, while for $\Omega_p \to 0$ there is a finite contribution from transverse magnetic modes. This contribution is due to the $m = 0$ term of the sum over Matsubara frequencies and it depends linearly upon T. It is seen that the force is attractive (it tends to contract the film) and it shows a maximum and a tail at the high plasma
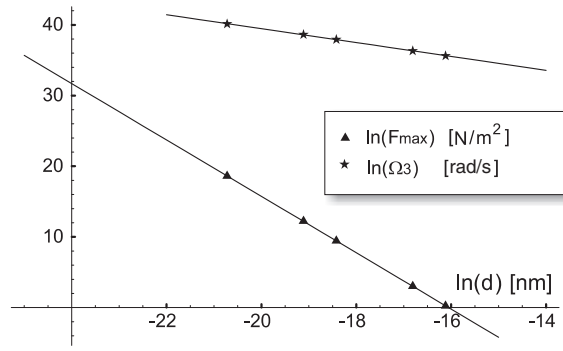
Figure 3. Maximum value of the force and the plasma frequency at which it occurs, as a function of distance: the fitting functions are $-64.05 - 3.98\ln(d)$ (triangles) and $19.83 - 0.98\ln(d)$ (stars).

frequency. As expected from the general behaviour of the electromagnetic-induced forces as a function of the distance, the maximum intensity reduces as a function of d, while its frequency moves to higher values. The dots in the figure correspond to the free-electron plasma frequency for sp-bonded simple metals. This should not be seen as an accurate prediction of the force value for metals. It indicates only that the force on real metal films may fall on both sides of the maximum, depending upon the film thickness.

Note that retardation effects are essential to obtain the maximum in the theoretical curve. This can be understood by a simple calculation of the force on a free standing metal film in the van der Waals (small d) regime at $T = 0°$ K. In this case, we have [2, 26]

$$F = -\frac{\hbar}{8\pi^2 d^3} \int_0^\infty \frac{(\epsilon_3(i\xi) - 1)^2}{(\epsilon_3(i\xi) + 1)^2}\, d\xi,\tag{5}$$

which leads to

$$F = -\frac{\hbar\Omega_s}{32\pi d^3},\tag{6}$$

with $\Omega_s = \Omega_3/\sqrt{2}$ frequency of the surface plasmon. Equation (6) does not show any maximum as a function of Ω_3. This is not surprising since the above expression is valid under the condition that d is much smaller than the plasma wavelength, therefore it is appropriate in the small plasma frequency regime only.

The behaviour of the maximum frequency as a function of d is given in figure 3: it is shown that in the range of thickness we have considered, the maximum frequency falls like d^{-1}, while the intensity maximum falls as d^{-4}, as expected for the interaction in the retarded regime. The behaviour of the force maximum, that is displaced to larger values for smaller thicknesses, can be understood by noticing that the attraction arises from the interaction between the surface plasmons at the two film boundaries [27–29]. At a given film thickness the interaction is screened by the electron gas with increasing efficiency as the plasma frequency increases. For large electron density $\Omega_3 \to \infty$, the force goes to zero and one surface does not feel the presence of the other. The maximum in the force results from the balance between the surface plasmons interaction and the screening effects. In particular, for small d a higher electron density is required to screen the attractive force.

Some interesting comments can be made on these data. The first concerns the unsupported film stability: the force tends to shrink the film and it has to be equilibrated by some repulsive interaction, most likely provided by the force built up by the valence electron rearrangement at the surfaces. Second, we note that the force can be tuned significantly by changing the
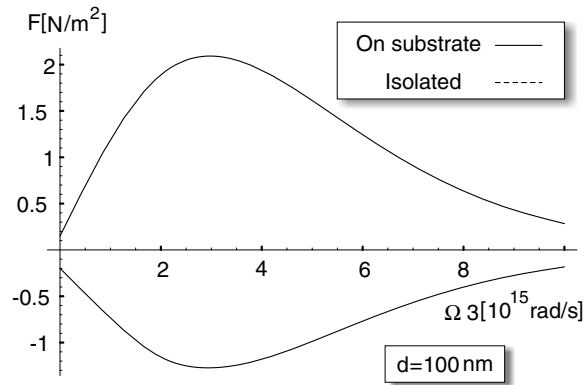
Figure 4. Force as a function of the film plasma frequency: a change in sign occurs when the isolated metallic film is placed on a perfectly reflecting substrate (ideal metal).

electron density of the metal: this effect could be useful in engineering the film properties for specific applications.

3. The film–ideal metal substrate interaction

To show how these conclusions are modified when the metal film is interacting with a substrate, we display in figure 4 the behaviour of the force per unit area on a film of $d = 100$ nm thickness deposited onto a perfectly reflecting substrate (corresponding to the configuration with $\epsilon_2 = 1$ and ϵ_1 equal to infinity), as a function of the film plasma frequency. This is a very simplified description of a bi-metallic interface, based on the assumption of the validity of the continuum model, which neglects all the details of the interactions between the atoms at the interface. It is expected to hold when the size of the film is large compared to the interface region (typically a few angstroms), so that the interface plays a minor role in determining the electromagnetic force. Note that the force becomes repulsive and nearly double in intensity, although it shows the same qualitative behaviour with a maximum and a long asymmetric tail at large frequency values. It comes from the difference between the electromagnetic force per unit area on the substrate side and that on the vacuum side the behaviour of the force can be understood by noticing that at $T = 0°$ K the exact calculation in the non-retarded limit gives the simple result

$$F(d) = \frac{\hbar \Omega_p}{32 \pi d^3} \sqrt{2} \tag{7}$$

showing the change of sign and the increased force value. This result is consistent with the behaviour of the London dispersion forces between dissimilar materials separated by a gap, which has been reported over many years [3, 30, 31] . In this case, the force is known to be repulsive when $\epsilon_1 \gtrless \epsilon_2 \gtrless \epsilon_3$ and attractive when $\epsilon_1 \gtrless \epsilon_2 \lessgtr \epsilon_3$ within a wide frequency range.

It is interesting to understand how the force between film boundaries in a multi-layer system is modified as a function of the film–substrate distance. For the case of a perfectly reflecting substrate, one can determine the range of distances over which the sign of the force changes. To this aim, one has to extend equation (2) to a configuration with more than three planar media. In practice, this amounts to replace the functions Q_{TM} and Q_{TE} by those appropriate to a multi-layer configuration. For a five-layer system the appropriate expressions were derived by Zhou and Spruch [32]:

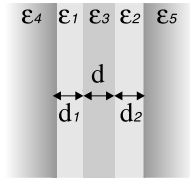$$Q_{TM} = Q_{TM1} Q_{TM2}, \qquad Q_{TE} = Q_{TE1} Q_{TE2}, \tag{8a}$$

Figure 5. Notation for five-layer system.

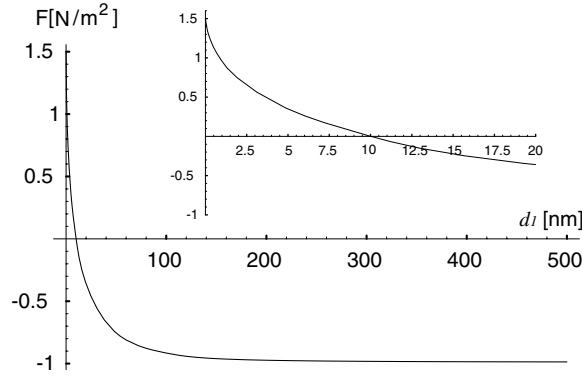**Figure 6.** Force on the film boundaries as a function of the film–substrate distance, the film plasma frequency is $\Omega_3 = 5 \times 10^{15}$ rad s^{-1}.

$$Q_{\text{TM1,TE1}} = \frac{\rho_{13}^{\text{TM,TE}} - \rho_{14}^{\text{TM,TE}} \, e^{-2\gamma_1 d_1}}{1 - \rho_{13}^{\text{TM,TE}} \rho_{14}^{\text{TM,TE}} \, e^{-2\gamma_1 d_1}} \, e^{-\gamma_3 d}, \tag{8b}$$

$$Q_{\text{TM2,TE2}} = \frac{\rho_{23}^{\text{TM,TE}} - \rho_{25}^{\text{TM,TE}} \, e^{-2\gamma_2 d_2}}{1 - \rho_{23}^{\text{TM,TE}} \rho_{25}^{\text{TM,TE}} \, e^{-2\gamma_2 d_2}} \, e^{-\gamma_3 d}, \tag{8c}$$

$$\rho_{mn}^{\text{TE}} = \frac{\gamma_m - \gamma_n}{\gamma_m + \gamma_n}, \qquad \rho_{mn}^{\text{TM}} = \frac{\gamma_m \epsilon_n - \gamma_n \epsilon_m}{\gamma_m \epsilon_n + \gamma_n \epsilon_m}, \tag{8d}$$

where γ_i is again given by (4) and the new indices refers to figure 5.

For the study of substrate–metal film interaction, we take ϵ_4 equal to infinity, $\epsilon_1 = \epsilon_2 = \epsilon_5 = 1$ while ϵ_3 is the metallic-film dielectric function (1). Since the configuration depends upon two parameters, the size d of the film and the film–substrate distance d_1, one can define the force F between the film boundaries, given by the derivative of the free energy with respect to d and the force F', obtained by deriving the free energy with respect to d_1, giving the interaction between the film and the substrate. Figure 6 shows the behaviour of the force F on the film boundaries as a function of the film–substrate distance for a 100 nm film. It is seen that the force remains constant and attractive if the distance d_1 is larger than the film thickness d; at lower distances the force decreases until it becomes repulsive. In other words, the film starts feeling a difference between the pressure from the metal substrate side and the external vacuum pressure, when the film–substrate distance is comparable with its thickness.

Discussions on device stability refer usually to the interaction between film and substrate (here we use the word substrate to indicate a structure of much larger size than the film, it could be a plate in a device), which gives rise to an attractive force F'. To show how this
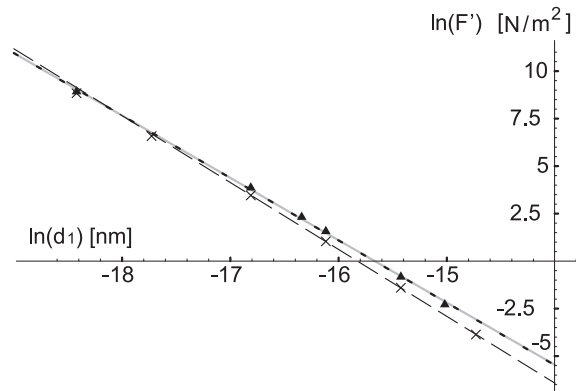
Figure 7. Film–substrate force as a function of the film–substrate distance between ideal metal substrate and real metal film, film thickness 100 nm (triangles) and 10 nm (crosses). The fitting functions are $-51.5 - 3.29 \ln(d_1)$ (dotted line) and $-55.7 - 3.52 \ln(d_1)$ (dashed line). The grey curve is the force between two semi-infinite systems, an ideal metal and a real metal with the same plasma frequency of the film.

interaction behaves as a function of the ideal film–substrate distance, we have calculated F' using equation (8d). It turns out to be attractive for any value of the film plasma frequency and, at distances smaller than the film size, it is considerably more intense than the force F on the film. This force is responsible for the change in sign observed in figure 4: if the film is close to the substrate, the difference between the attractive force on the film boundaries tends to stretch the film, causing a repulsive force between them.

The behaviour of the film–substrate force F' in the range of distances d_1 below the film thickness, where the substrate effect is more significant, is illustrated by the results shown in figure 7 for a 100 nm film with the plasma frequency $\Omega_3 = 5 \times 10^{15}$ rad s^{-1} and a perfectly reflecting substrate.

Note that in this range of distances the force F' increases like d_1^{-x} with $3 < x < 4$ (the simple d_1^{-4} behaviour at all distances is the characteristic of the interaction between ideal metal plates only and it is appropriate for real metals only at large distances). At 100 nm distance this force is approximately -4.8 N m^{-2} (the Casimir force between ideal metals at the same distance is of the order of -10 N m^{-2}), while the force on the film boundaries is approximately -1 N m^{-2}. The grey curve in the figure displays the calculated force per unit area for a semi-infinite metal interacting with a perfectly reflecting semi-infinite substrate. It can be seen that it does not deviate significantly from the curve for the 100 nm film. At higher distances the attractive force decreases while the force on the film remains approximately constant.

We report in the same figure the calculated F' for a 10 nm film: in this case the force versus distance behaviour is rather different, showing a significantly higher exponent than in the 100 nm case (3.52 rather than 3.29). Clearly, this behaviour cannot be understood using arguments based on results for semi-infinite systems: for a semi-infinite metal interacting with an ideal substrate one would expect the exponent x to become closer to 3 upon decreasing the distance. The fact that it results to be significantly higher is a direct consequence of the finite thickness of the film. Indeed, as first pointed out by Zhou and Sprunch [32] higher negative exponents characterize the interaction in the presence of film of very small thickness. An important consequence of this behaviour is that the calculated F' at 10 nm
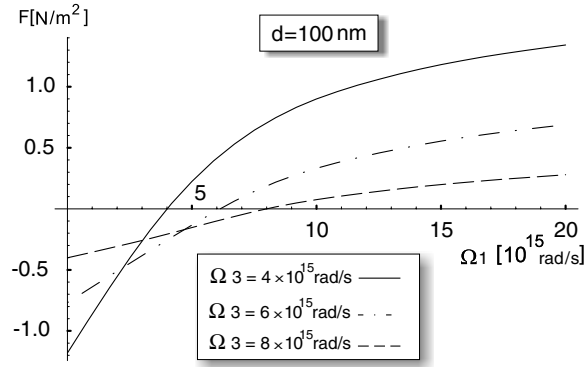
Figure 8. Force on the film boundaries as a function of the substrate plasma frequency calculated for different film plasma frequencies, in the calculation about two thousands Matsubara frequencies have been used.

distance (approximately -7511.7 N m^{-2}) is considerably higher than the force F on the film boundaries (approximately 0.001 N m^{-2}).

We can conclude that the interaction of a metal film with a perfectly reflecting substrate leads to an attractive film–substrate force and, at short distances, to a repulsive force on the film boundaries. For 50–100 nm thick films these forces are approximately of the same order when the film–substrate distance is comparable with the film size. In the low-distance range (1–10 nm) the force on the film can be neglected and the attractive film–substrate interaction prevails in intensity. These considerations are expected to be important for systems, such as micro-switches, that consist of two conducting electrodes, where one is fixed and the other one is able to move, being suspended by a mechanical spring. The stability of the system may depend upon the electromagnetic-induced force acting on the mobile film [33, 34].

4. The bimetallic interfaces

The situation changes if we consider a more realistic description of the substrate. Referring to figure 1 this corresponds to take $\epsilon_1 = 1 - \Omega_1^2/\omega^2$. Figure 8 shows the behaviour of the force per unit area on a 100 nm metal film deposited onto various metal substrates as a function of the substrate plasma frequency. Note that the force is attractive when $\Omega_1 < \Omega_3$ and is repulsive in the opposite case. For $\Omega_1 \gg \Omega_3$ we get the repulsive force corresponding to a perfectly reflecting substrate. The change in the sign it can be easily understood by considering the force in the small d-limit, i.e. in the non-retarded regime. At $T = 0°$ K the force calculated from equation (5) is simply given by

$$F = \frac{\hbar}{32\pi d^3} \frac{\Omega_s \left(\Omega_1^2 - \Omega_3^2\right)}{\bar{\Omega}(\bar{\Omega} + \Omega_s)}, \tag{9}$$

where

$$\bar{\Omega} = \sqrt{\left(\Omega_1^2 + \Omega_3^2\right)/2} \tag{10}$$

is the interface plasmon frequency obtained from the relation $\epsilon_1(\omega) = -\epsilon_3(\omega)$. Note that F shows the expected change from the repulsive to the attractive behaviour.

Figure 9 shows that the curves of the force on films deposited onto different substrates as a function of the film plasma frequencies. The curves show two extrema: on the repulsive side a maximum, which increases in intensity and moves to a higher frequency upon increasing
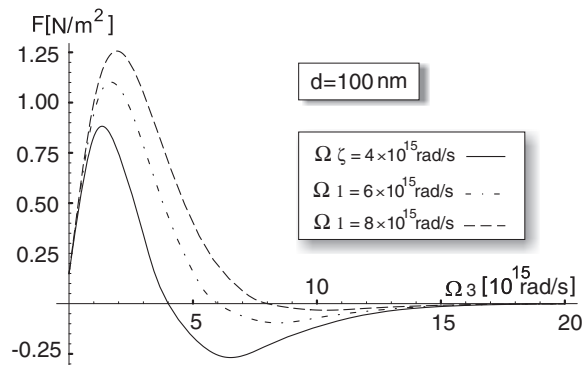
Figure 9. Force on the film boundaries as a function of the film plasma frequency calculated for different substrate frequencies, in the calculation about two thousands Matsubara frequencies have been used.
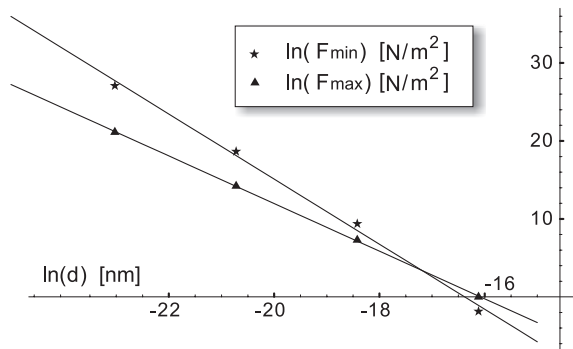

Figure 10. $F(\Omega_3)$ minimum and $F(\Omega_3)$ maximum as a function of distance d, the fitting functions are, respectively, $-68.36 - 4.17\ln(d)$ (stars) and $-49.10 - 3.05\ln(d)$ (triangles), calculations were performed at fixed $\Omega_1 = 5 \times 10^{15}$ rad s^{-1}.

the substrate plasma frequency; on the attractive side a minimum which decreases upon increasing Ω_1 and shifts to higher frequency values. This behaviour is consistent with the previous conclusions concerning the ideal substrate: as the plasma frequency Ω_1 increases the repulsive force on the film becomes dominant.

It is interesting to see how the extrema behave upon varying the film thickness. As shown in figure 10, the intensity of the repulsive maximum falls like d^{-3}, in the range of distances we are considering, while for the attractive minimum it falls approximatively as d^{-4}. Indeed the occurrence of the maximum can be understood on the basis of the short-distance formula (9), which gives a d^{-3} dependence of the force, while the behaviour of the attractive part is mainly due to retarded interactions. These results lead to the conclusion that the electromagnetic-fluctuation-induced forces can give a contribute of opposite sign, and with different dependence upon the film size, to the deposited film stability.

As in the case of the ideal substrate, we can study the electromagnetic-fluctuation-induced force F_1 between the film and the substrate as a function of the film–substrate distance. Based on the previous analysis we expect the film–substrate force to be attractive and lead to a repulsive or attractive force between the film boundaries depending upon the difference between the plasma frequencies: if $\Omega_1 \gg \Omega_3$ the situation is similar to the ideal substrate
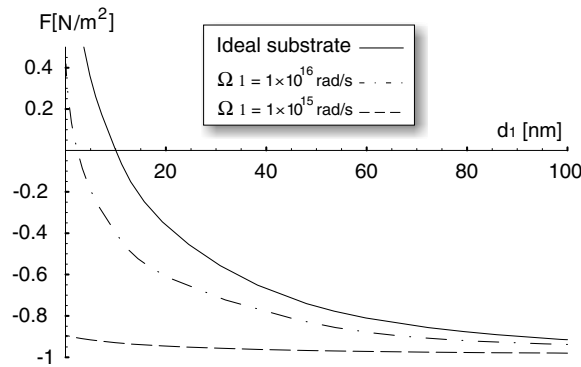
Figure 11. Film boundaries force as a function of film–substrate distance, comparison between an ideal substrate (continuous line) and a real metal substrate with the plasma frequencies 10^{16} rad s^{-1} (dot-dashed line) and 10^{15} rad s^{-1} (dashed line). The film plasma frequency is 5×10^{15} rad s^{-1}.
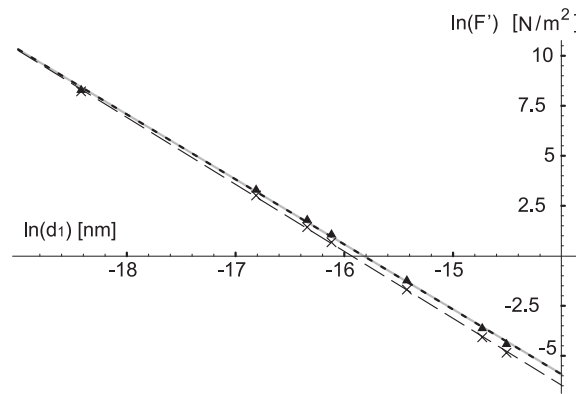

Figure 12. Film–substrate force as a function of the film–substrate distance for real metals with 5×10^{15} rad s^{-1}, film thicknesses 100 nm (triangles) and 10 nm (crosses). The fitting functions are $-51.4 - 3.25 \ln(d_1)$ (dotted line) and $-53.3 - 3.35 \ln(d_1)$ (dashed line). The grey curve is the force between two semi-infinite bulks, two real metals with the same plasma frequency.

case, while for $\Omega_1 \ll \Omega_3$ the force on the film is only weakly modified by the interaction. The various cases are illustrated in figure 11.

To see how our results depend upon the film thickness we have compared the calculated curves for film–substrate force as a function of the distance d_1 with the electromagnetic-induced bulk–bulk interaction. We have compared the bulk–bulk interaction with film–substrate interaction for typical values of the plasma frequency. As shown in figure 12 the results seem not to depend significantly upon the film thickness for 100 nm films, while size effects become important for d of the order of 10 nm.

It is clear, from these calculations that, in the nanometric distances range, the adoption of the simple force expression appropriate to ideal plates is not correct. Both the sign and the intensity of the force may result wrong, if material properties and thickness effects are not properly accounted in the theory.

5. Discussion and conclusions

We have presented a rather complete set of results based on a continuum dielectric model to illustrate trends in the behaviour of the electromagnetic-fluctuation-induced forces on free-standing and supported metal films, which allow us to identify the conditions under which the force is attractive or repulsive and how it depends upon the film thickness and the interacting substrate (plate) properties.

We have shown that both the sign and the intensity of the force between a film and a plate depend upon the difference in the plasma frequencies and can be modified upon changing the carrier density. This is in line with the recent proposal of modulating the Casimir force between a metal and a semiconductor plate by illuminating the semiconducting material, i.e. by enhancing the electron plasma and creating a hole plasma in the semiconductor plate [35]. We expect that any experimental system that allows us to change the difference in plasma frequencies can be used to modulate the electromagnetic force.

An adequate description of the electronic properties in thin film does, in general, require consideration of the changes in the electron energy levels resulting from the confinement of the electrons. Since these effects are observed for film thicknesses of several nanometers, one can argue that the results of the present paper may be significantly modified if quantum size effects are taken into account. To clarify this point we have calculated the dielectric permittivity of metallic films adopting the particle in a box model [36], in which independent electrons are confined by a surface potential of a given length scale d along the z-direction, with the eigenvalue spectrum

$$E_{\mathbf{k},n} = \frac{\hbar^2 \mathbf{k}^2}{2m} + E_0 n^2 \qquad n = 1, 2, 3 \ldots, \tag{11}$$

where $\mathbf{k}$ is a two-dimensional wavevector and $E_0 = \hbar^2 \pi^2 / 2md^2$. The electron confinement leads to the quantization of the transverse component of the momentum and formation of lateral sub-bands. The surface effects are built in the eigenstates:

$$\psi_{\mathbf{k},n}(\mathbf{r}) = \sqrt{\frac{2}{A \times d}} \sin\left(\frac{n\pi}{d} z\right) e^{i\mathbf{k}\cdot\rho}, \tag{12}$$

where $A \times d$ is the film volume and ρ is the positive vector in the xy-plane. Under such conditions the film dielectric tensor is given by

$$\epsilon_{\alpha,\alpha}(\omega) = 1 - \frac{\Omega_p^2}{\omega^2} - \frac{8\pi\, e^2}{A \times d}\,\frac{1}{m^2\omega^2} \sum_{\mathbf{k},n} \sum_{\mathbf{k}',n'} f_0(\epsilon_{\mathbf{k},n}) \frac{(\epsilon_{\mathbf{k},n} - \epsilon_{\mathbf{k}',n'})|\langle\psi_{\mathbf{k},n}|\hat{p}_\alpha|\psi_{\mathbf{k}',n'}\rangle|^2}{(\epsilon_{\mathbf{k},n} - \epsilon_{\mathbf{k}',n'})^2 - (\hbar\omega)^2}. \tag{13}$$

This expression differs from the model dielectric function in several respects: (i) it has a tensor character with $\epsilon_{x,x} = \epsilon_{y,y} \neq \epsilon_{z,z}$, (ii) the plasma frequency Ω_p depends upon the film density, which, at a fixed chemical potential at $T = 0°$ K, changes as a function of the film thickness, (iii) it accounts for transitions between lateral sub-bands (the Fermi momentum being constant the number of occupied sub-band increases upon increasing the film size). It can be easily shown that these transitions do not affect the lateral components of the dielectric tensor. They modify the low-frequency behaviour of $\epsilon_{z,z}$. The model has been used to interpret optical and transport properties of thin films [37–40].

We have calculated the electromagnetic-induced forces on free-standing metals films of different Fermi energy. Figure 13 shows the results of a calculation for a 50 nm film. One can compare them with those plotted in figure 3. It can be noted that, although the value of the force is modified by the inclusion of size effects, the behaviour as a function of the free-electron plasma frequency remains the same. Similar results have been obtained in other
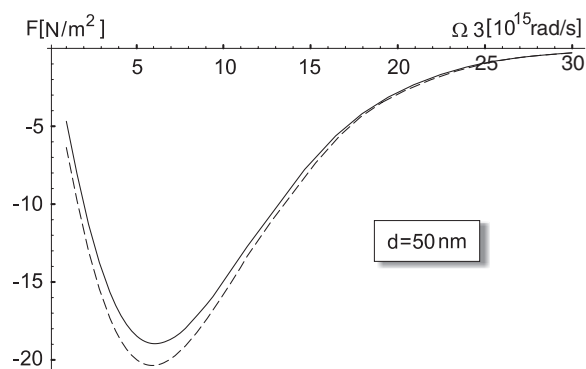
Figure 13. Force as a function of the film plasma frequency: comparison between Drude model (dashed line) and particle in a box model (continuous line).

cases and will be reported elsewhere, in a more detailed study of quantum size effects on electromagnetic-induced forces.

We conclude that the main trend of the results given in the present paper is not modified by quantum size effects for thickness above 10 nm.

The present theory can be improved along two main lines. Inclusion of bulk relaxation effects, both in the continuum dielectric theory and in the particle in a box model, is expected to modify the calculated value of the force. A more accurate description of surface electromagnetic field, which treats the modifications to the Fresnel optics caused by the surface, may also lead to appreciable changes specially for film size of the order of few nanometers.

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